Class XI Session 2025-26
Subject - Mathematics
Sample Question Paper - 6

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. If sin α + sin β = a and cos α - cos β = b, then tan $\frac{\alpha-\beta}{2}$ = **[1]**

a) $\sqrt{a^2+b^2}$ b) $\sqrt{a^2-b^2}$

c) $-\frac{a}{b}$ d) $-\frac{b}{a}$

2. Let A be the finite set containing n distinct elements. The number of relations that can be defined on A is **[1]**

a) 2^n b) n^2

c) 2^{n^2} d) 2^{n-1}

3. One of the two events must occur. If the chance of one is $\frac{2}{3}$ of the other, then odds in favour of the other are **[1]**

a) 2 : 3 b) 3 : 1

c) 3 : 2 d) 1 : 3

4. $\frac{d}{dx}\left(\sec^{-1} x\right)$ is equal to **[1]**

a) $\frac{1}{|x|\sqrt{x^2-1}}$ for |x| > 1 b) $\frac{1}{1+x^2}$

c) $\frac{1}{3x\sqrt{x^2-x}}$ for |x| > 1 d) $\frac{-1}{x\sqrt{x^2-1}}$ for |x| > 1

5. The distance of the point P (1, – 3) from the line 2y – 3x = 4 is **[1]**

a) $3\sqrt{13}$ b) 13

c) $\sqrt{13}$ d) $\frac{7}{13}\sqrt{13}$

6. If A and B are two sets then $A \cap (A \cap B') = \ldots.$ **[1]**

a) $\in$ b) ϕ

c) B d) A

7. Mark the correct answer for $\left(\frac{1-i}{1+i}\right)^2$ = ? **[1]**

a) 1 b) $\frac{1}{\sqrt{2}}$

c) $\frac{-1}{2}$ d) -1

8. If A = {x $\in$ Z : 0 $\leq$ x $\leq$ 12} and R is the relation in A given by R = {(a, b) : |a - b| is a multiple of 4}. Then, the set of all elements related to 1 is **[1]**

a) {1, 3, 9} b) {1, 4, 6}

c) {1, 5, 9} d) {2, 4, 6}

9. If |x + 2| $\leq$ 9, then **[1]**

a) $x \in (-\infty, -7) \cup (11, \infty)$ b) x $\in$ (- 7 , 11)

c) x $\in$ [- 11 , 7] d) $x \in (-7, -\infty) \cup [\infty, 11)$

10. The value of sin 78° - sin 66° - sin 42° + sin 60° is **[1]**

a) -1 b) $\frac{1}{2}$

c) $\frac{-1}{2}$ d) None of these

11. Two finite sets have m and n elements respectively. The total number of subsets of first set is 56 more than the total number of subsets of the second set. The values of m and n respectively are. **[1]**

a) 8, 7 b) 5, 1

c) 6, 3 d) 7, 6

12. If x, 2y, 3z are in A.P., where the distinct numbers x, y, z are in G.P. then the common ratio of the G.P. is **[1]**

a) 3 b) $\frac{1}{2}$

c) 2 d) $\frac{1}{3}$

13. If the coefficient of x in $\left(x^2 + \frac{\lambda}{x}\right)^5$ is 270, then $\lambda =$ **[1]**

a) 6 b) 5

c) 3 d) 4

14. If x and b are real numbers . If b $>$ 0 and |x| $> b$, then **[1]**

a) $x \in (-\infty, -b) \cup (b, \infty)$ b) $x \in [-\infty, b)$

c) $x \in (-b, b)$ d) $x \in (-b, \infty)$

15. The number of non-empty subsets of the set {1, 2, 3, 4} is: **[1]**

a) 14 b) 16

c) 17 d) 15

16. (tan 15° tan 25° tan 45° tan 65° tan 75°) = ? **[1]**

a) 3

b) 2

c) 1

d) $\sqrt{3}$

17. If z is any complex number, then $\frac{z-\bar{z}}{2i}$ is **[1]**

a) purely real

b) purely imaginary

c) either 0 or purely imaginary

d) either 0 or purely real

18. If ${}^{15}P_{r-1} : {}^{16}P_{r-2} = 3 : 4$, then r = ? **[1]**

a) 10

b) 12

c) 14

d) 8

19. **Assertion(A):** In the expansion $(x + x^{-2})^n$ the coefficient of eighth term and nineteenth term are equal, then n = 25. **[1]**

Reason (R): Middle term in the expansion of $(x + a)^n$ has the greatest binomial coefficient.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17 is 3. **[1]**

Reason (R): The mean deviation about the mean for the data 38, 70, 48, 40, 42, 55, 63, 46, 54, 44 is 8.5.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. if A = {1, 3, 5} and B = {2, 3}, then show that $A \times B \neq B \times A$. **[2]**

OR

Write the relations as the sets of ordered pairs: A relation R on the set {0, 1, 2, ..., 10} defined by 2x + 3y = 12.

22. Differentiate $\left(\frac{x^2+5x-6}{4x^2-x+3}\right)$. **[2]**

23. Find the equation of the ellipse in the cases: eccentricity e = $\frac{1}{2}$ and major axis = 12. **[2]**

OR

Find the vertex, focus, axis, directrix and latus-rectum of the following parabolas $4x^2 + y = 0$.

24. If A = {a, b, c, d, e}, B = {a, c, e, g} and C = {b, e, f, g}, verify that: $B \cup C = C \cup B$ **[2]**

25. State whether the two lines are parallel, perpendicular or neither: Through (3, 15) and (16, 6); through (-5, 3) and (8, 2) **[2]**

Section C

26. Let $f : R \to R$: $f(x) = x^3$ for all $x \in R$. Find its domain and range. Also, draw its graph. **[3]**

27. Solve the system of linear inequation: $3x - x > x + \frac{4-x}{3} > 3$ **[3]**

28. Verify that (-1, 2, 1), (1, -2, 5), (4, -7, 8) and (2, -3, 4) are the vertices of a parallelogram. **[3]**

OR

Show that the points (-2, 3, 5), (1, 2, 3) and (7, 0, -1) are collinear.

29. The coefficient of $\frac{1}{x^{17}}$ in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is **[3]**

OR

Using g binomial theorem, expand $\left\{(x+y)^5 + (x-y)^5\right\}$ and hence find the value of $\{(\sqrt{2}+1)^5 + (\sqrt{2}-1)^5\}$

30. Write the argument of $(1+\sqrt{3})$ (1 + i) (cos θ + i sin θ). **[3]**

OR

Find the square root of -15 - 8i.

31. Using the properties of sets and their complements prove that: A - (B ∪ C) = (A - B) ∩ (A - C) **[3]**

Section D

32. A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that all the three balls are red. **[5]**

33. Evaluate the following limits: $\lim\limits_{x \to \sqrt{10}} \frac{\sqrt{7+2x}-(\sqrt{5}+\sqrt{2})}{x^2-10}$. **[5]**

OR

Differentiate $\frac{\sin x}{x}$ from first principle.

34. If S be the sum, P be the product and R be the sum of reciprocals of n terms in a G.P, prove that $P^2 = \left(\frac{S}{R}\right)^n$. **[5]**

35. Prove that: tan 20° tan 30° tan 40° tan 80° = 1 **[5]**

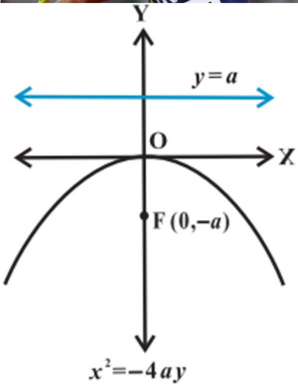
OR

Prove that: sin 20° sin 40° sin 80° = $\frac{\sqrt{3}}{8}$

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Indian track and field athlete Neeraj Chopra, who competes in the Javelin throw, won a gold medal at Tokyo Olympics. He is the first track and field athlete to win a gold medal for India at the Olympics.



i. Name the shape of path followed by a javelin. If equation of such a curve is given by x^2 = -16y, then find the coordinates of foci. (1)

ii. Find the equation of directrix and length of latus rectum of parabola x^2 = -16y. (1)

iii. Find the equation of parabola with Vertex (0,0), passing through (5,2) and symmetric with respect to y-axis and also find equation of directrix. (2)

OR

Find the equation of the parabola with focus (2, 0) and directrix x = -2 and also length of latus rectum. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

A teacher conducted a surprise test of Mathematics, Physics and Chemistry for class XI on Monday.

The mean and standard deviation of marks obtained by 50 students of the class in three subjects are given below:

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40.9
Standard deviation	12	15	20

i. Which of the three subjects shows the highest variability? (1)

ii. What is the coefficient of variation of marks obtained by the students in Chemistry? (1)

iii. What is the coefficient of variation of marks obtained by the students in Physics? (2)

OR

What is the coefficient of variation of marks obtained by the students in Mathematics? (2)

38. A permutation is **an act of arranging the objects or numbers in order**. Combinations are the way of selecting the objects or numbers from a group of objects or collections, in such a way that the order of the objects does not matter. **[4]**

PERMUTATIONS

In, how many ways can the letters of the word PERMUTATIONS be arranged if the

(i) words start with P and end with S

(ii) vowels are all together

Solution

Section A

1.

(d) $-\frac{b}{a}$

Explanation:

Given: sin α + sin β = a ......(i)

cos α - cos β = b .....(ii)

Dividing (i) by (ii):

$\Rightarrow \frac{\sin\alpha+\sin B}{\cos\alpha-\cos B} = \frac{a}{b}$

$\Rightarrow \frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)} = \frac{a}{b}$ [$\because$ sin A + sin B = 2 sin $\left(\frac{A+B}{2}\right)$ cos $\left(\frac{A-B}{2}\right)$ and cos A + cos B = -2 sin $\left(\frac{A+B}{2}\right)$ sin $\left(\frac{A-B}{2}\right)$]

$\Rightarrow \frac{\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{-\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)} = \frac{a}{b}$

$\Rightarrow$ cot $\left(\frac{\alpha-\beta}{2}\right)$ = $-\frac{a}{b}$

$\Rightarrow \frac{1}{\cot\left(\frac{\alpha-\beta}{2}\right)} = \frac{1}{-\frac{a}{b}}$

$\Rightarrow$ tan $\left(\frac{\alpha-\beta}{2}\right)$ = $-\frac{b}{a}$

2.

(c) 2^{n^2}

Explanation:

Number of relations that can be defined on A = 2^{n^2}

3.

(c) 3 : 2

Explanation:

Let P(B) = x

Then, P(A) = 2 $\times$ 3

P(A) + P(B) = x + 2 $\times$ 3 = 5 $\times$ 3

$\Rightarrow$ 5 $\times$ 3 = 1 ($\because$ They are exhaustive events)

$\Rightarrow$ x = 35

Now, P(A) = 25 and PB = 35

$\therefore$ Odd in favour of B = $\frac{3}{51}$ - $\frac{3}{5}$ = 32 = 3 : 2

4. **(a)** $\frac{1}{|x|\sqrt{x^2-1}}$ for |x| > 1

Explanation:

y = $\sec^{-1}$ x = sec y = x

$\Rightarrow$ y' = cot y cos y

$\Rightarrow \frac{1}{|x|\sqrt{x^2-1}}$ for |x| > 1

5.

(c) $\sqrt{13}$

Explanation:

We know that the distance of the point P (1, – 3) from the line 2y – 3 x – 4 = 0 is the length of perpendicular from the point to the line which is given by

$\left|\frac{2(-3)-3-4}{\sqrt{13}}\right| = \sqrt{13}$

6.

(d) A

Explanation:

$(A \cap B') = A$

$\Rightarrow A \cap (A \cap B') = A \cap A = A$

7.

(d) -1

Explanation:

$\frac{(1-i)}{(1+i)} = \frac{(1-i)}{(1+i)} \times \frac{(1-i)}{(1-i)} = \frac{(1-i)^2}{(1-i^2)} = \frac{1+i^2-2i}{(1+1)} = \frac{1-1-2i}{2} = \frac{-2i}{2}$ = -i

$\Rightarrow \left(\frac{1-i}{1+i}\right)^2$ = $(-i)^2 = i^2$ = -1

8.

(c) {1, 5, 9}

Explanation:

The set of elements related to 1 is {1, 5, 9}.

Since, |1 - 1| = 0 is multiple of 4

|5 - 1| = 4 is a multiple of 4

and |9 - 1| = 8 is a multiple of 4

9.

(c) x ∈ [- 11 , 7]

Explanation:

$|x + 2| \leq 9$

$\Rightarrow -9 \leq x + 2 \leq 9 \quad [\because |x| \leq a \Leftrightarrow -a \leq x \leq a]$

$\Rightarrow -9 - 2 \leq x + 2 - 2 \leq 9 - 2$

$\Rightarrow -11 \leq x \leq 7$

$x \epsilon [-11, 7]$

10.

(d) None of these

Explanation:

sin 78^o - sin 66^o - sin 42^o + sin 60^o

= sin 78^o - sin 42^o - sin 66^o + sin 60^o

= 2 sin $\left(\frac{78^\circ - 42^\circ}{2}\right)$ cos $\left(\frac{78^\circ + 42}{2}\right)$ - sin 66^o + sin 60^o [∵ sin A - sin B = 2 sin $\left(\frac{A-B}{2}\right)$ cos $\left(\frac{A+B}{2}\right)$]

= 2 sin 18^o cos 60^o - sin 66^o + sin 60^o

= 2 $\times$ $\frac{1}{2}$ sin 18^o - sin 66^o + $\frac{\sqrt{3}}{2}$

= sin 18^o - sin 66^o + $\frac{\sqrt{3}}{2}$

= 0.309 - 0.914 + 0.866

= 0.261

11.

(c) 6, 3

Explanation:

Since, let A and B be such sets, i.e., n (A) = m, and n(B) = n

Thus, n (P(A)) = 2^m , n (P(B)) = 2^n

Therefore, n (P(A)) – n (P(B)) = 56, i.e., $2^m - 2^n = 56$

$\Rightarrow 2^n (2^{m-n} - 1) = 2^3\, 7$

$\Rightarrow n = 3, 2^{m-n} - 1 = 7$

$\Rightarrow m = 6$

12.

(d) $\frac{1}{3}$

Explanation:

It is given that x, 2y, 3z are in A.P

$\therefore$ 2y - x = 3z - 2y

$\Rightarrow$ 2y + 2y = x + 3z

$\Rightarrow$ 4y = x + 3z

$\Rightarrow$ x = 4y - 3z ...(i)

and it is also given that x, y, z are in G.P

$\therefore$ Common ratio $r = \frac{y}{x} = \frac{z}{y}$ ...(ii)

$y \times y = x \times z$

$\Rightarrow y^2 = xz$ ...(iii) substituting the value of x = 4y - 3z in eq. (iii), we obtain

$y^2 = (4y - 3z)(z)$

$\Rightarrow y^2 = 4yz - 3z^2$

$\Rightarrow 3z^2 - 4yz + y^2 = 0$

$\Rightarrow 3z^2 - 3yz - yz + y^2 = 0$

$\Rightarrow$ 3z(z - y) - y(z - y) = 0

$\Rightarrow$ (3z - y)(z - y) = 0

$\Rightarrow$ 3z - y = 0 & z - y = 0

$\Rightarrow$ 3z = y & z = y but z and y are distinct numbers

$\Rightarrow z = \frac{1}{3}y$ & $z \neq y$

$\Rightarrow \frac{z}{y} = \frac{1}{3}$

$\Rightarrow r = \frac{1}{3}$ [from eqn. (ii)]

Therefore, the correct option is $\frac{1}{3}$.

13.

(c) 3

Explanation:

The coefficient of x in the given expansion where x occurs at the (r + 1)th term.

We have

${}^5C_r\, (x^2)^{5-r} \left(\frac{\lambda}{x}\right)^r$

$= {}^5C_r \lambda^r x^{10-2r-r}$

For it to contain x, we must have:

10 - 3r = 1

$\Rightarrow$ r = 3

$\therefore$ the required coefficient of x in the given expansion:

${}^5C_3 \lambda^3 = 10\lambda^3$

Now, we have

$10\lambda^3 = 270$

$\Rightarrow \lambda^3 = 27$

$\Rightarrow \lambda = 3$

14. **(a)** $x \in (-\infty, -b) \cup (b, \infty)$

Explanation:

We have $|x| > a \Leftrightarrow x < -a$ or x > a

So |x| $> b \Rightarrow x < -b$ or x > b

$\Rightarrow x \in (-\infty, -b) \cup (b, \infty)$

15.

(d) 15

Explanation:

Total no. of subset including empty set = 2^n

So total subset = 2^4 = 16

The no. of non empty set = 16 - 1 = 15

16.

(c) 1

Explanation:

Given exp. = (tan 15° tan 75°) (tan 25° tan 65°) tan 45°

= (tan 15° cot 15°) (tan 25° cot 25°) $\times$ 1 = 1.

17. **(a)** purely real

Explanation:

Let z = x + iy

Then $\bar{z}$ = x - iy

$\therefore$ z - $\bar{z}$ = (x + iy) - (x - iy) = 2iy

Now $\frac{z-\bar{z}}{2i}$ = y

Hence $\frac{z-\bar{z}}{2i}$ is purely real.

18.

(c) 14

Explanation:

$\frac{{}^{15}P_{r-1}}{{}^{16}P_{r-1}} = \frac{3}{4}$

$\Rightarrow \frac{15!}{(15-(r-1))!} \times \frac{(16-(r-2))!}{16!} = \frac{3}{4}$

$\Rightarrow \frac{1}{16} \times \frac{(18-r)!}{(16-r)!} = \frac{3}{4}$

$\Rightarrow$ (18 - r) (17 - r) = 12

$\Rightarrow$ r^2 - 35r - 294 = 0

$\Rightarrow$ (r - 21) (r - 14) = 0

$\Rightarrow$ r = 14 $[\because r \leq 16]$

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

It is given that coefficients of $T_8 = T_{19}$

$T_{r+1} = {}^nC_r a^{n-r} b^r$

Applying to the above question we get

${}^nC_7 = {}^nC_{18}$

Now ${}^nC_r = {}^nC_{n-r}$

Using the formula we get

n - 7 = 18

n = 25

It is also true that the middle term has the greatest coefficient in the expansion of x + a)^n since in pascal's triangle the middle term has the largest value.

20.

(c) A is true but R is false.

Explanation:

Assertion Mean of the given series

$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$

$= \frac{4+7+8+9+10+12+13+17}{8}$ = 10

xi	|xi - $\bar{x}$|
4	|4 - 10| = 6
7	|7 - 10| = 3
8	|8 - 10| = 2
9	|9 - 10| = 1
10	|10 - 10| = 0
12	|12 - 10| = 2
13	|13 - 10| = 3
17	|17 - 10| = 7
$\sum x_i$ = 80	$\sum \|x_i - \bar{x}\|$ = 24

$\therefore$ Mean deviation about mean

$= \frac{\Sigma|x_i - \bar{x}|}{n} = \frac{24}{8}$ = 3

Reason Mean of the given series

$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$

$= \frac{38+70+48+40+42+55}{+63+46+54+44}$ = 50

$\therefore$ Mean deviation about mean

$= \frac{\Sigma|x_i - \bar{x}|}{n}$

$= \frac{84}{10}$ = 8.4

Hence, Assertion is true and Reason is false.

Section B

21. Given, = $\{1, 3, 5\}$ and B = $\{2, 3\}$.

Now, A×B = {1,3, 5} × {2, 3}

$\therefore$ A×B = $\{(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)\}$ ...(i)

and B×A = {2, 3} × {1, 3, 5}

$\therefore$B×A = $\{(2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5)\}$ ....(ii)

From eq. (i) and (ii), we get

A×B $\neq$ B×A

Hence proved.

OR

Given, (x, y) R 2x + 3y = 12

where x and y {0, 1, 2, …, 10}

2x + 3y = 12

$\Rightarrow$ 2x = 12 – 3y

$\Rightarrow x = \frac{12-3y}{2}$

Put y = 0,

$\Rightarrow x = \frac{12-3(0)}{2} = \frac{12}{2} = 6$

Put y = 2,

$\Rightarrow x = \frac{12-3(2)}{2} = \frac{12-6}{2} = \frac{6}{2} = 3$

Put y = 4,

$\Rightarrow x = \frac{12-3(4)}{2} = \frac{12-12}{2} = \frac{0}{2} = 0$

$\therefore$ R = {(0, 4), (3, 2), (6, 0)}

22. By quotient rule of differentiation,we have,

$\frac{d}{dx}\left(\frac{x^2+5x-6}{4x^2-x+3}\right)$

$= \frac{(4x^2-x+3)\cdot\frac{d}{dx}(x^2+5x-6)-(x^2+5x-6)\cdot\frac{d}{dx}(4x^2-x+3)}{(4x^2-x+3)^2}$

$= \frac{(4x^2-x+3)(2x+5)-(x^2+5x-6)(8x-1)}{(4x^2-x+3)^2} = \frac{(9+54x-21x^2)}{(4x^2-x+3)^2}$

23. Given that, e = $\frac{1}{2}$ and major axis = 12

i.e., 2a = 12 or a = 6

We have e $= \sqrt{1-\frac{b^2}{a^2}}$

$\Rightarrow \frac{1}{2} = \sqrt{1-\frac{b^2}{36}}$

Squaring both sides, we get;

$\frac{1}{4} = \frac{36-b^2}{36}$

$\Rightarrow$ 36 = 144 - b^2

$\Rightarrow$ b^2 = 27

Substituting the values of a^2 and b^2, we get;

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{36} + \frac{y^2}{27} = 1$

$\Rightarrow \frac{3x^2+4y^2}{108} = 1$

$\Rightarrow$ $3x^2 + 4y^2 = 108$

Which is the required equation of the ellipse.

OR

We are given that:

$4x^2 + y = 0$

$\Rightarrow \frac{-y}{4} = x^2$

Comparing the given equation with $x^2 = -4ay$

4a = $\frac{1}{4}$ a = $\frac{1}{16}$

$\therefore$ Vertex = (0, 0)

Focus = (0, -a) = $\left(0, \frac{-1}{16}\right)$

Equation of the directrix:

y = a

i.e. y = $\frac{1}{16}$

Axis = x = 0

Therefore, length of the latus rectum = 4a = $\frac{1}{4}$

24. To prove: B ∪ C = C ∪B

Since the element of set C is not provided,

Suppose x be any element of C

L.H.S = B ∪ C

= { a, c, e, g } ∪ { x |x ∈ C}

= { a, c, e, g, x}

= {x, a, c, e, g}

= { x |x ∈ C} ∪ { a, c, e, g }

= C ∪ B

= R.H.S

Hence proved.

25. Through (3,15) and (16, 6); through (-5, 3) and (8, 2)

Suppose m_1 be the slope of the line joining (3, 15) and (16, 6) and m_2 be the slope of the line joining (-5, 3) and (8, 2).

$\therefore$ $m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{6-15}{16-3} = \frac{-9}{13}$ and $m_2 = \frac{y_2-y_1}{x_2-x_1} = \frac{2-3}{8+5} = \frac{-1}{13}$

Now, $m_1m_2 = \frac{-9}{13} \times \frac{-1}{13} = \frac{9}{169}$

Since, $m_1m_2 \neq -1$ and $m_1 \neq m_2$.

Hence, the given lines are neither parallel nor perpendicular.

Section C

26. We have, f :R $\rightarrow$ R: f(x) = x^3 for all x $\in$ R

dom (f) = R and range (f) = R.

We have,

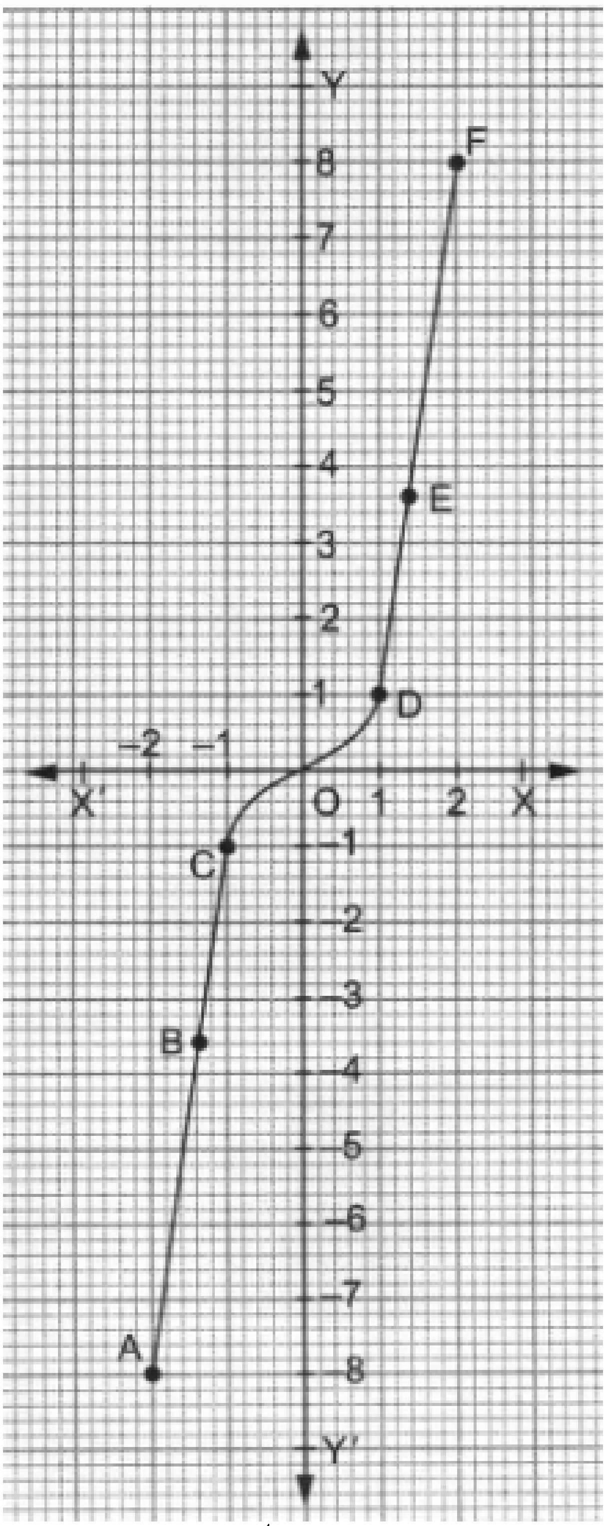
x	-2	-1.5	- 1	0	1	1.5	2
f(x) = x^3	-8	-3.375	- 1	0	1	3.375	8

On a graph paper, we draw X' OX and YOY' as the x-axis and the y-axis respectively.

We take the scale as 5 small divisions = 1 unit.

Now, we plot the points A(-2, - 8), B(-1.5, -3.375), C(-1, -1), 0(0, 0), D(l, 1), E(1.5, 3.375) and F(2, 8).

We join these points freehand successively to obtain the required curve shown in the figure below.



27. Given that $3x - x > x + \frac{4-x}{3} > 3$

When, $3x – x > x + \frac{4-x}{3}$

$\Rightarrow 2x > x + \frac{4-x}{3}$

Subtracting x from both the sides in above equation

$\Rightarrow 2x – x > x + \frac{4-x}{3} - x$

$\Rightarrow x > \frac{4-x}{3}$

Multiplying both the sides by 3 in the above equation

$\Rightarrow 3x > 3(\frac{4-x}{3})$

$\Rightarrow 3x > 4 – x$

Adding x on both the sides in above equation

$\Rightarrow 3x + x > 4 -x + x$

$\Rightarrow 4x > 4$

Dividing both the sides by 4 in the above equation

$\Rightarrow \frac{4x}{4} > \frac{4}{4}$

$\Rightarrow$ x > 1

Now when, x + $\frac{4-x}{3}$> 3

Multiplying both the sides by 3 in above equation

$\Rightarrow$ 3x + 3($\frac{4-x}{3}$)> 3(3)

$\Rightarrow$ 3x + 4 – x > 9

$\Rightarrow$ 2x + 4 > 9

Subtracting 4 from both the sides in above equation

$\Rightarrow$ 2x + 4 – 4 > 9 – 4

$\Rightarrow$ 2x > 5

Dividing both the sides by 2 in above equation

$\Rightarrow \frac{2x}{2} > \frac{5}{2} \Rightarrow x > \frac{5}{2}$

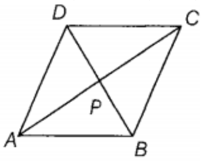
Merging overlapping intervals

$x > \frac{5}{2}$ & x > 1

Therefore,

$x \in \left(\frac{5}{2}, \infty\right)$

28. Let A (-1, 2, 1), B(1, - 2, 5), C(4, - 7, 8) and D (2, - 3 , 4) are the vertices of a quadrilateral ABCD.



Then, mid-point of

$AC = \left(\frac{-1+4}{2}, \frac{2-7}{2}, \frac{1+8}{2}\right) = \left(\frac{3}{2}, \frac{-5}{2}, \frac{9}{2}\right)$ $\left[\because \text{ coordinates of mid-point } \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)\right]$

Similarly, mid-point of BD $= \left(\frac{3}{2}, -\frac{5}{2}, \frac{9}{2}\right)$

Mid-points of both the diagonals are the same (i.e., they bisect each other).

Hence, ABCD is a parallelogram.

OR

Let A(-2, 3, 5), B (1, 2, 3) and C(7, 0, -1) be three given points.

Then $AB = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2} = \sqrt{9+1+4} = \sqrt{14}$

$BC = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} = \sqrt{36+4+16} = \sqrt{56} = 2\sqrt{14}$

$AC = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2} = \sqrt{81+9+36} = \sqrt{126} = 3\sqrt{14}$

Now AC = AB + BC

Therefore,A,B,C are collinear.

29. We quad have the general term of (x + a)^n is $T_{r+1} = {}^nC_r \quad (x)^{n-r}a^r$

Now consider $\left(x^4 - \frac{1}{x^3}\right)^{15}$

Here $T_{r+1} = {}^{15}C_r \quad (x^4)^{15-r}\left(-\frac{1}{x^3}\right)^r$

Comparing the indices of x in x^{-17} and in T_{r+1}, we get 60 - 4r - 3r = -17 $\Rightarrow$ 7r = 77 $\Rightarrow$ r = 11

Therefore the required term is $T_{11+1} = T_{12} = {}^{15}C_{11} \quad (x^4)^4\left(-\frac{1}{x^3}\right)^{11}$

Hence the coefficient of $\frac{1}{x^{17}} = -{}^{15}C_{11}$ = -1365

OR

We have

$(x+y)^5 + (x-y)^5 = 2\left[{}^5C_0\, x^5 + {}^5C_2\, x^3y^2 + {}^5C_4\, x^1y^4\right]$

$= 2\left(x^5 + 10x^3y^2 + 5xy^4\right)$

Putting $x = \sqrt{2}$ *and* $y = 1$, *we get*

$(\sqrt{2}+1)^5 + (\sqrt{2}-1)^5 = 2\left[(\sqrt{2})^5 + 10(\sqrt{2})^3 + 5\sqrt{2}\right]$

$= 2\left[4\sqrt{2} + 20\sqrt{2} + 5\sqrt{2}\right]$

$= 58\sqrt{2}$

30. Let the argument of $(1+i\sqrt{3})$ be α. Then

$\tan\alpha = \frac{\sqrt{3}}{1} = \tan\frac{\pi}{3}$

$\Rightarrow \alpha = \frac{\pi}{3}$

Let the argument of (1 + i) be β. Then,

$\tan\beta = \frac{1}{1} = \tan\frac{\pi}{4}$

$\Rightarrow \beta = \frac{\pi}{4}$

Let the argument of $(\cos\theta + i\sin\theta)$ be γ. Then,

$\tan\gamma = \frac{\sin\theta}{\cos\theta} = \tan\theta$

$\Rightarrow \gamma = \theta$

$\therefore$ The argument of

$(1+i\sqrt{3})(1+i)\ (\cos\theta + i\sin\theta) = \alpha + \beta + \gamma = \frac{\pi}{3} + \frac{\pi}{4} + \theta = \frac{7\pi}{12} + \theta$ [$\arg(z_1)(z_2) = \arg(z_1) + \arg(z_2)$]

Hence, the argument of

$(1+i\sqrt{3})(1+i)\ (\cos\theta + i\sin\theta)$ is $\frac{7\pi}{12} + \theta$

OR

Let $x + yi = \sqrt{-15-8i}$

Squaring both sides, we get

$(x + yi)^2 = -15 - 8i$

$x^2 - y^2 + 2xyi = -15 - 8i$

Comparing the real and imaginary parts

$x^2 - y^2 = -15$ .... (i)

$2xy = -8 \Rightarrow xy = -4$

Now, we know that

$(x^2 + y^2)^2 = (x^2 - y^2) + 4x^2y^2$

$= (-15)^2 + 4(-4)^2$

= 225 + 64

= 289

$\therefore x^2 + y^2 = 17$ ..... (ii) [Neglecting (-) sign as $x^2 + y^2 > 0$]

Solving (i) and (ii), we get

$x = \pm 1$ and $y = \pm 4$

Since the sign of xy is (-)

$\therefore$ x = 1, y = -4

And x = -1, y = 4

$\therefore \sqrt{-15-8i} = \pm(1-4i)$

31. Let x ∈ {A - (B ∪ C)}

x ∈ A and x ∉ (B ∪ C)

x ∈ A and (x ∉ B and x ∉ C)

(x ∈ A and x ∉ B) and (x ∈ A and x ∉ C)

x ∈ (A - B) and x ∈ (A - C)

x ∈ {(A - B) ∩ (A - C)}

A - (A - B) ⊆ (A - B) ∩ (A - C) ...(i)

Again, let

y ∈ (A - B) ∩ (A - C)

y ∈ (A - B) and y ∈ (A - C)

(y ∈ A and y ∉ B) and (y ∈ A and y ∉ C)

y ∈ A and y ∉ B ∪ C

y ∈ {A - (B ∪ C)}

(A - B) ∩ (A - C) ⊆ A - (B ∪ C) ...(ii)

From eqs. (i) and (ii)

A - (B ∪ C) = (A - B) ∩ (A ∪ C) Hence proved

Section D

32. Given that the bag contains 13 balls and three balls are drawn from the bag

So, the total number of ways of drawing three balls = number of total outcomes $= n(S) = {}^{13}C_3$

Now, we have to find the probability that all three balls drawn are red,

Let A be the event that all drawn balls are red

There are 8 red balls in the bag

So, number of favourable outcomes i.e. all three balls are red = n(A) = ${}^{8}C_3$

We know that,

P(A) = $\frac{\mathbf{n(A)}}{\mathbf{n(S)}} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

P(all the three balls are red) = $\frac{{}^{8}C_3}{{}^{13}C_3}$

$= \frac{\frac{8!}{3!(8-3)!}}{\frac{13!}{3!(13-3)!}} \left[\because {}^{n}C_r = \frac{n!}{(n-r)!r!}\right]$

$= \frac{\frac{8\times7\times6\times5!}{3!5!}}{\frac{13\times12\times11\times10!}{3\times2\times1\times10!}}$

$= \frac{\frac{8\times7\times6}{3\times2}}{13\times2\times11}$

$= \frac{28}{143}$

33. We have to find the value of $\lim\limits_{x\to\sqrt{10}} \frac{\sqrt{7+2x}-(\sqrt{5}+\sqrt{2})}{x^2-10}$

Re-writing the equation as

$= \lim\limits_{x\to\sqrt{10}} \frac{\sqrt{7+2x}-\sqrt{(\sqrt{5}+\sqrt{2})^2}}{x^2-10}$

$= \lim\limits_{x\to\sqrt{10}} \frac{\sqrt{7+2x}-\sqrt{5+2+2\sqrt{10}}}{x^2-10}$

$= \lim\limits_{x\to\sqrt{10}} \frac{\sqrt{7+2x}-\sqrt{7+2\sqrt{10}}}{x^2-10}$

Now rationalizing the above equation

$= \lim\limits_{x\to\sqrt{10}} \frac{(\sqrt{7+2x}-\sqrt{7+2\sqrt{10}})}{x^2-10} \frac{(\sqrt{7+2x}+\sqrt{7+2\sqrt{10}})}{(\sqrt{7+2x}+\sqrt{7+2\sqrt{10}})}$

Formula: (a + b) (a - b) = $a^2 - b^2$

$= \lim\limits_{x\to\sqrt{10}} \frac{(7+2x-(7+2\sqrt{10}))}{x^2-10} \frac{(1)}{(\sqrt{7+2x}+\sqrt{7+2\sqrt{10}})}$

$= \lim\limits_{x\to\sqrt{10}} \frac{(2x-2\sqrt{10})}{x^2-10} \frac{(1)}{(\sqrt{7+2x}+\sqrt{7+2\sqrt{10}})}$

$= \lim\limits_{x\to\sqrt{10}} \frac{2(x-\sqrt{10})}{(x+\sqrt{10})(x-\sqrt{10})} \frac{(1)}{(\sqrt{7+2x}+\sqrt{7+2\sqrt{10}})}$

$= \lim\limits_{x\to\sqrt{10}} \frac{2(1)}{(x+\sqrt{10})(1)} \frac{(1)}{(\sqrt{7+2x}+\sqrt{7+2\sqrt{10}})}$

Now we can see that the indeterminant form is removed, so substituting x as $\sqrt{10}$

$= \frac{2}{2\sqrt{10}} \frac{1}{(2\sqrt{7+2\sqrt{10}})}$

$= \frac{1}{2\sqrt{10}} \frac{1}{(\sqrt{7+2\sqrt{10}})}$

OR

Let $f(x)=\frac{\sin x}{x}$

By using first principle of derivative,

$f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\therefore f'(x)=\lim\limits_{h\to 0}\frac{\frac{\sin(x+h)}{x+h}-\frac{\sin x}{x}}{h}$

$=\lim\limits_{h\to 0}\frac{x\sin(x+h)-(x+h)\sin x}{x(x+h)\times h}$

$=\lim\limits_{h\to 0}\frac{x[\sin(x+h)-\sin x]-h\sin x}{h\cdot x(x+h)}$

$=\lim\limits_{h\to 0}\frac{x\left[2\cdot\cos\left(\frac{x+h+x}{2}\right)\cdot\sin\left(\frac{x+h-x}{2}\right)\right]-h\sin x}{h\cdot x(x+h)}$

$\left[\because \sin C-\sin D=2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$

$=\lim\limits_{h\to 0}\frac{x\left[2\cdot\sin\frac{h}{2}\cdot\cos\left(x+\frac{h}{2}\right)\right]-h\sin x}{h\cdot x(x+h)}$

$=\lim\limits_{\frac{h}{2}\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\cdot\lim\limits_{h\to 0}\frac{\cos\left(x+\frac{h}{2}\right)}{(x+h)}-\lim\limits_{h\to 0}\frac{\sin x}{x(x+h)}$

$=(1)\cdot\frac{\cos x}{x}-\frac{\sin x}{x^2}\left[\because \lim\limits_{x\to 0}\frac{\sin x}{x}=1\right]$

$=\frac{\cos x}{x}-\frac{\sin x}{x^2}$

34. Let G. P. be a, ar, ar^2, …..

Where r < 1

$S=\frac{a(1-r^n)}{1-r}$

$R=\frac{1}{a}+\frac{1}{ar}+--+n$

$=\frac{\frac{1}{a}\left[\left(\frac{1}{r}\right)^n-1\right]}{\frac{1}{r}-1}\ [\because r<1 \text{ then } \frac{1}{r}>1$

$=\frac{1}{a}\cdot\frac{1-r^n}{r^n}\cdot\frac{r}{1-r}$

$=\frac{1-r^n}{ar^{n-1}(1-r)}$

P = a.ar.ar^2 ... ar^{n-1}

= an.$r^{1+2+----+(n-1)}$

$=a^n.r^{\frac{n(n-1)}{2}}$

$=a^n.r^{\frac{n(n-1)}{2}}$

L.H.S. = P^2R^n

$=a^{2n}.r^{n(n-1)}\cdot\frac{(1-r^n)^n}{ar^{n-1}(1-r)^n}$

= S^n

$P^2=\left(\frac{S}{R}\right)^n$

Hence proved.

35. LHS = tan 20° tan 30° tan 40° tan 80°

$=\frac{1}{\sqrt{3}}(\tan 20^\circ\tan 40^\circ\tan 80^\circ)\left[\because \tan 30^\circ=\frac{1}{\sqrt{3}}\right]$

$=\frac{(\sin 20^\circ\sin 40^\circ\sin 80^\circ)}{(\cos 20^\circ\cos 40^\circ\cos 80^\circ)\sqrt{3}}$

$=\frac{(2\sin 20^\circ\sin 40^\circ)\sin 80^\circ}{\sqrt{3}(2\cos 20^\circ\cos 40^\circ)\cos 80^\circ}$

Applying

$\Rightarrow$ 2 sin A sin B = cos (A - B) - cos (A + B) and 2 cos A cos B = cos(A + B) + cos (A - B), we get

$=\frac{[\cos(40^\circ-20^\circ)-\cos(20^\circ+40^\circ)]\sin 80^\circ}{[\cos(20^\circ+40^\circ)+\cos(40^\circ-20^\circ)]\cos 80^\circ\sqrt{3}}$

$=\frac{(\cos 20^\circ-\cos 60^\circ)\sin 80^\circ}{\sqrt{3}(\cos 60^\circ+\cos 20^\circ)\cos 80^\circ}$

$=\frac{\left(\cos 20^\circ-\frac{1}{2}\right)\sin 80^\circ}{\sqrt{3}\left(\frac{1}{2}+\cos 20^\circ\right)\cos 80^\circ}$

$=\frac{2\cos 20^\circ\sin 80^\circ-\sin 80^\circ}{\sqrt{3}(\cos 80^\circ+2\cos 20^\circ\cos 80^\circ)}$

Now,

$\Rightarrow$ 2 sin A cos B = sin (A + B) + sin (A - B)

$= \frac{\sin(80^\circ+20^\circ)+\sin(80^\circ-20^\circ)-\sin 80^\circ}{\sqrt{3}[\cos 80^\circ+\cos(20^\circ+80^\circ)+\cos(80^\circ-20^\circ)]}$

$= \frac{\sin 100^\circ+\sin 60^\circ-\sin 80^\circ}{\sqrt{3}(\cos 80^\circ+\cos 100^\circ+\cos 60^\circ)}$

$= \frac{\sin 100^\circ+\sin 60^\circ-\sin(180^\circ-100^\circ)}{\sqrt{3}(\cos 80^\circ+\cos(180^\circ-80^\circ)+\cos 60^\circ)}$

$= \frac{\sin 100^\circ+\frac{\sqrt{3}}{2}-\sin 100^\circ}{\sqrt{3}(\cos 80^\circ-\cos 80^\circ+\cos 60^\circ)}$

$= \frac{\frac{\sqrt{3}}{2}}{\sqrt{3}\left(\frac{1}{2}\right)}$ = 1 = RHS

OR

Given, LHS $= sin20^o sin40^o sin80^o$

= $\frac{1}{2}$ [2 sin 20° · sin 40°] sin 80^o [multiplying and dividing by 2]

= $\frac{1}{2}[cos(20^o - 40^o) - cos(20^o + 40^o)] \cdot \sin 80^\circ$ [$\because 2\sin x \cdot \sin y$ = cos (x - y) - cos (x + y)]

= $\frac{1}{2}$ $[cos(-20^o) - cos60^o]sin80^o$

= $\frac{1}{2}$ [cos 20^o - $\frac{1}{2}$]· sin 80° [$\because$ cos (- θ) = cos θ and cos 60^o = $\frac{1}{2}$]

= $\frac{1}{2} \times \frac{1}{2}\left[2\left(\cos 20^\circ - \frac{1}{2}\right) \cdot \sin 80^\circ\right]$ [again multiplying and dividing by 2]

= $\frac{1}{4}[2\cos 20^\circ \cdot \sin 80^\circ - \sin 80^\circ]$

= $\frac{1}{4}[sin(20^o + 80^o) - sin(20^o - 80^o) - sin80^o]$ [$\because 2\cos x \cdot \sin y = sin(x+y) - sin(x-y)$]

= $\frac{1}{4}$ $[sin100^o - sin(-60^o) - sin80^o]$

= $\frac{1}{4}$ [sin 100^o + sin 60^o - sin 80^o] [$\because$ sin (- θ) = - sin θ]

= $\frac{1}{4}$ [sin (180^o - 80^o) + sin 60^o - sin 80^o] [$\because$ sin 100^o = sin (180^o - 80^o)]

= $\frac{1}{4}$ [sin 80^o + sin 60^o - sin 80^o] [$\because$ sin (π − θ) = sin θ]

= $\frac{1}{4}$ × sin 60^o = $\frac{1}{4}$ × $\frac{\sqrt{3}}{2}$ [$\because$ sin 60^o = $\frac{\sqrt{3}}{2}$]

= $\frac{\sqrt{3}}{8}$ = RHS

Hence proved.

Section E

36. i. The path traced by Javelin is parabola. A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane.

compare x^2 = -16y with x^2 = -4ay

$\Rightarrow$ - 4a = -16

$\Rightarrow$ a = 4

coordinates of focus for parabola x^2 = -4ay is (0, -a)

$\Rightarrow$ coordinates of focus for given parabola is (0, -4)

ii. compare x^2 = -16y with x^2 = -4ay

$\Rightarrow$ -4a = -16

$\Rightarrow$ a = 4

Equation of directrix for parabola x^2 = -4ay is y = a

$\Rightarrow$ Equation of directrix for parabola x^2 = -16y is y = 4

Length of latus rectum is 4a = 4 × 4 = 16

iii. Equation of parabola with axis along y - axis

x^2 = 4ay

which passes through (5, 2)

$\Rightarrow$ 25 = 4a × 2

$\Rightarrow$ 4a = $\frac{25}{2}$

hence required equation of parabola is

$x^2 = \frac{25}{2}y$

$\Rightarrow$ $2x^2$ = 25y

Equation of directrix is y= -a

Hence required equation of directrix is 8y + 25 = 0.

OR

Since the focus (2,0) lies on the x-axis, the x-axis itself is the axis of the parabola.

Hence the equation of the parabola is of the form either $y^2 = 4ax$ or $y^2 = -4ax$.

Since the directrix is x = -2 and the focus is (2,0), the parabola is to be of the form $y^2 = 4ax$ with a = 2.

Hence the required equation is $y^2 = 4(2)x = 8x$

length of latus rectum = 4a = 8

37. i. The subject with greater C.V. is more variable than others.

Therefore, the highest variability in marks is in Chemistry.

ii. Standard deviation of Chemistry = 20

C.V. (in Chemistry) = $\frac{20}{40.9} \times 100 = 48.89$

iii. Standard deviation of Physics = 15

The coefficient of variation, C.V. = $\frac{\text{Standard deviation}}{Mean} \times 100$

C.V. (in Physics) = $\frac{15}{32} \times 100 = 46.87$

OR

Standard deviation of Mathematics = 12

The coefficient of variation, C.V. = $\frac{\text{Standard deviation}}{Mean} \times 100$

C.V. (in Mathematics) = $\frac{12}{42} \times 100 = 28.57$

38. Total letters in the word PERMUTATIONS = 12.

Here T = 2

(i) Now first letter is P and last letter is S which are fixed.

So the remaining 10 letters are to be arranged between P and S.

$\therefore$ Number of Permutations

$= \frac{10!}{2!} = \frac{10\times9\times8\times7\times6\times5\times4\times3\times2!}{2!} = 1814400$

(ii) There are 5 vowels in the word PERMUTATIONS. All vowels can be put together.

$\therefore$ Number of permutations of all vowels together $= {}^5P_5$

$= \frac{5!}{0!} = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Now consider the 5 vowels together as one letter. So the number of letters in the word when all vowels are together = 8.

$\therefore$ Number of Permutations $= \frac{8!}{2!} = \frac{8\times7\times6\times5\times4\times3\times2!}{2!} = 20160$

Hence the total number of permutations $= 120 \times 20160 = 2419200$